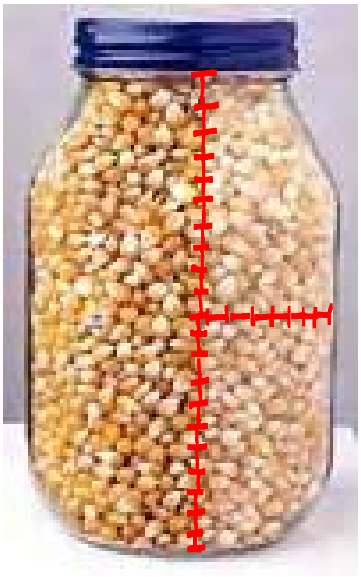
Sec 1.1 -Analyzing Numerical Data
Fermi Problems: Estimating Large Numbers

Name: _____

Practice Problems:

1. **The jar's inner dimensions of the jar are approximately a cylinder with a height of 16 cm and a radius of 4.5 cm. The jar completely filled to the top with popcorn kernels.**



$V_{\text{cylinder}} = \pi r^2 h = \pi(4.5)^2(16) \approx 1018\ cm^3$

What would be a good lower bound for the number of kernels in the jar and why? THE JAR IS AT LEAST 20 KERNELS HIGH AND HAS A RADIUS OF 7 KERNELS. SO IF WE LET THE LONGEST DIMENSION OF A KERNEL BE THE UNIT OF MEASURE, WE COULD ESTIMATE A LOWERBOUND TO BE: $V = \pi(7)^2(20) \approx 3079\ \text{Kernels}^3$

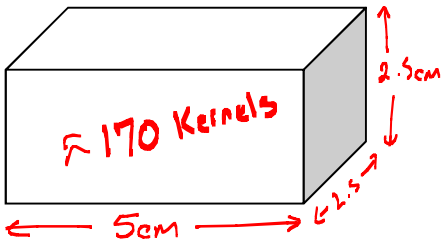
POWER OF TEN
1000 KERNELS

What would be a good upper bound for the number of kernels in the jar and why? AT MOST, THE JAR IS 30 KERNELS HIGH WITH A RADIUS OF 10 KERNELS. SO AGAIN IF WE LET THE SHORTEST DIMENSION BE THE UNIT OF MEASURE, WE COULD ESTIMATE AN UPPERBOUND TO BE: $V = \pi(10)^2(30) \approx 9424\ \text{Kernel}^3$

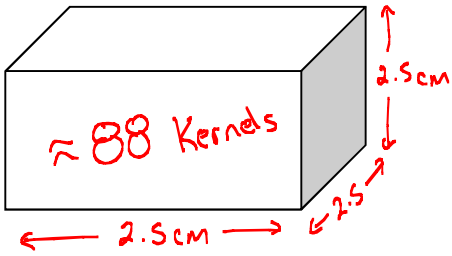
POWER OF 10
10,000 KERNELS

Use at least 2 different containers

Write the dimension on the drawing below for the container you used for solid #1.



Write the dimension on the drawing below for the container you used for solid #2.



Fill out the following table:	*Solid #1*	*Solid #2*
Volume = $l \cdot w \cdot h$	$(5cm)(2.5cm)(2.5cm) = 31.25\ cm^3$	$(2.5cm)(2.5cm)(2.5cm) = 15.625\ cm^3$
Number of Kernels	170	88
Ratio : EITHER RATIO COULD BE USED AS LONG AS IT USED CONSISTENTLY.	$\frac{\text{KERNELS}}{cm^3} = \frac{170}{31.25} = 5.44\ \frac{\text{KERNEL}}{cm^3}$	$\frac{\text{KERNEL}}{cm^3} = \frac{88}{15.625} \approx 5.632\ \frac{\text{KERNEL}}{cm^3}$
	$\frac{cm^3}{\text{KERNEL}} = \frac{31.25}{170} \approx 0.184\ \frac{cm^3}{\text{KERNEL}}$	$\frac{cm^3}{\text{KERNEL}} = \frac{15.625}{88} \approx 0.178\ \frac{cm^3}{\text{KERNEL}}$

AVG RATIO ≈ 5.5

AVG RATIO $\approx .181$

Using the information from the table what would be a good estimate for the number of kernels in the large jar?

ABOUT 5,600 KERNELS

IF WE USE KERNELS/cm^3:

$\frac{\text{KERNELS}}{cm^3} : \frac{5.5}{1} = \frac{x}{1018}$ ← VOLUME OF JAR

$x \approx 5599$

(Bonus Question) After kernels are popped they take up approximately 32 times more space than they did when they were kernels. In the movie "Real Genius", the students wanted revenge with a professor and filled his house with kernels which later popped. If the professor's house contained approximately 22,000 cubic feet (or 622,970,625 cm^3), how many kernels would be needed to do this?

VOLUME OF KERNEL PACKED = $0.181\ \frac{cm^3}{\text{KERNEL}}$

VOLUME OF POPPED KERNEL = $32(0.181) \approx 5.8\ \frac{cm^3}{\text{POPCORN}}$

$\frac{622970625\ cm^3}{5.8\ cm^3} \approx 107,408,000$ KERNELS

2. **The jar's inner dimensions of the jar are approximately a cylinder with a height of 18 cm and a radius of 7 cm. The jar completely filled to the top with jelly beans.** $V_{\text{cyl}} = \pi r^2 h = \pi(7)^2(18) \approx 2771\ cm^3$

What would be a good lower bound for the number of jelly beans in the jar and why?

THE JAR IS AT LEAST 11 JELLY BEANS TALL WITH A RADIUS OF 4 JELLYBEANS. IF WE LET THE LONGEST DIMENSION OF A JELLY BEAN BE THE UNIT OF MEASURE THEN A LOWER BOUND ESTIMATE COULD BE:

$V = \pi(4)^2(11) \approx 553\ \text{jelly bean}^3$

LOWERBOUND POWER 10

100

What would be a good upper bound for the number of jelly beans in the jar and why?

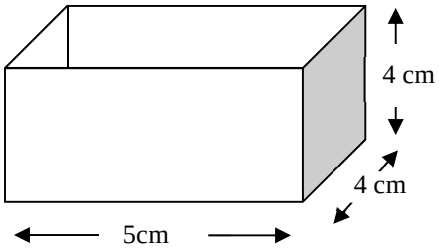
THE JAR IS AT MOST 20 JELLYBEANS TALL WITH A RADIUS OF 7. IF WE LET THE SMALLEST DIMENSION OF A JELLYBEAN BE THE UNIT OF MEASURE THEN AN UPPERBOUND ESTIMATE COULD BE:

$V = \pi(7)^2(20) \approx 3079\ \text{jellybean}^3$

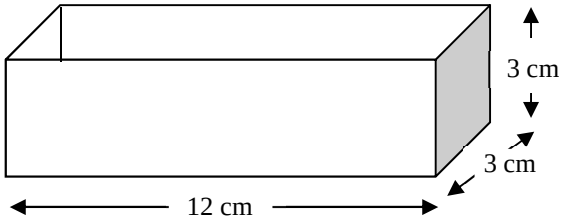
UPPERBOUND POWER OF 10

10,000

A few students conducted a study of some solids and found the following container (solid #1) held 32 jelly beans.



A few students conducted a study of some solids and found the following container (solid #1) held 43 jelly beans.



Fill out the following table:

	Solid #1	*Solid #2*
Volume $= l \cdot w \cdot h$	$(5cm)(4cm)(4cm) = 80\,cm^3$	$(12cm)(8cm)(3cm) = 108\,cm^3$
Number of Jelly Beans	32	43
Ratio EITHER RATIO CAN BE USED AS LONG AS YOU ARE CONSISTENT.	$\frac{\text{JELLY BEAN}}{cm^3} = \frac{32}{80} = 0.4\ \frac{\text{JELLY BEAN}}{cm^3}$ $\frac{cm^3}{\text{JELLY BEAN}} = \frac{80}{32} = 2.5\ \frac{cm^3}{\text{JELLY BEAN}}$	$\frac{\text{JELLY BEAN}}{cm^3} = \frac{43}{108} \approx 0.398\ \frac{\text{JELLY BEAN}}{cm^3}$ $\frac{cm^3}{\text{JELLY BEAN}} = \frac{108}{43} \approx 2.512\ \frac{cm^3}{\text{JELLY BEAN}}$

Using the information from the table what would be a good estimate for the number of jellybeans in the large jar?

RATIO: $\frac{\text{JELLY BEAN}}{cm^3} : \frac{0.4}{1} = \frac{x}{2771}$ ← VOLUME OF JAR

$x = 1108.4$ JELLY BEANS

$\approx$ 1108 JELLY BEANS

3. **A person just purchased the vending machine shown. Each compartment has the dimensions of 5 inches by 7 inches by 17 inches. Assuming the vending machine uses Gumballs that are approximately spherical and 1 inch in diameter, how many gumballs should fit in one of the compartments?**
(Hint: Packing spheres in a rectangular prism usually take up 190% of the volume of the spheres.)
SHOW YOUR WORK

① VOLUME OF GUMBALL $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.5)^3 \approx 0.524\ in^3$

② PACKING SPACE PER GUMBALL $= 1.9 \times$ VOLUME $= 1.9 * 0.524 \approx 0.996\ in^3$

③ VOLUME OF CONTAINER $= l \cdot w \cdot h = (5)(7)(17) = 595\ in^3$

④ NUMBER OF GUMBALLS $= \frac{595\ in^3}{0.996\ in^3} \approx 597$ GUMBALLS

≈ 597

4. **A stock person at Wal-Mart has decided that they would like to completely fill the ball bin in the toy section with one type of ball. The ball the stock person decided to use had a radius of 16 cm. The ball bin is 2.1 meters tall and the opening at the top is 1.5 meters by 1.5 meters. How many balls should fit inside the bin?**
(Hint: Packing spheres in a rectangular prism usually take up 190% of the volume of the spheres.)
SHOW YOUR WORK

① VOLUME OF BALL $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(16)^3 \approx 17157.3\ cm^3$

② PACKING SPACE PER BALL $= 1.9 \times$ VOLUME $= 1.9 * 17157.3$
$= 32598.9\ cm^3$

③ VOLUME OF CONTAINER $= (210\ cm)(150\ cm)(150\ cm)$
$= 4725000\ cm^3$

④ # OF BALLS $= \frac{4725000\ cm^3}{32598.9\ cm^3} \approx 145$ BALLS

≈ 145

5. **A football field is 360 feet long and 160 feet wide. The principal is making an evacuation plan for the school. How many students can the principal expect to fit on the football field in an emergency? (Remember the expected floor space a standing person occupies is about 2.5 sq feet)**
SHOW YOUR WORK

AREA OF FIELD $= (360)(160) = 57600\ ft^2$

OF PEOPLE $= \frac{57600}{2.5} = 23{,}040$ PEOPLE

23,040

6. **If every person in the world were to own an equal share of the earth's land, how many square miles would each person have? You may assume the earth is a sphere with a radius is 3959 miles and that 20 percent of its surface is land suitable to live on. There are approximately 7.2 billion people on earth as of 2014.**
SHOW YOUR WORK

SURFACE AREA $\bigcirc = 4\pi r^2$

SURFACE AREA OF EARTH $= 4\pi(3959)^2 \approx 196961284.3\ miles^2$

SUITABLE LAND AREA $= (0.2)(196961284.3) = 39392256.9\ miles^2$

$\frac{\text{Square miles}}{\text{Person}}: \frac{39392256.9}{7200000000} \approx .00547$ sq miles per person

$\approx .00547\ \frac{miles^2}{person}$

WHICH IS ABOUT 152527 sq ft per person (A 390 ft BY 390 ft square)

7. **Explain what steps and assumptions you might take to approximate how many tables are in our school?**

- SAMPLE A FEW CLASSROOMS AND DETERMINE AN AVERAGE NUMBER OF TABLES PER CLASSROOM
- FIND A MAP AND MULTIPLE THE MEAN NUMBER OF TABLES PER CLASSROOM BY THE NUMBER OF CLASSROOMS.

- SAMPLE A FEW CLASSROOMS TO FIND THE AVERAGE NUMBER OF TABLES PER SQUARE FOOT.
- USING A MAP ESTIMATE THE TOTAL SQUARE FEET OF CLASSROOM SPACE AND MULTIPLY IT BY THE MEAN NUMBER OF TABLES PER SQUARE FOOT.

8. How many combinations are possible on a 4 number computer cable lock. Each space can be any number 0 – 9. The only exception is that all four numbers cannot be the same. How many combinations are possible?

$10 \cdot 10 \cdot 10 \cdot 10 - 10$

(0-9 for each of the four spaces)

THERE ARE 10 RESTRICTIONS (0000, 1111, ..., 9999)

9990 COMBINATIONS

9. The country of Scitsitats requires the people in their country to have license tags on their car such that the first 3 characters are English letters but no letter may repeat. The last 3 characters must each be a number 0 – 9 and again no numbers can be repeated. How many license tags are possible?

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8$

11,232,000

11 MILLION

10. Social Security numbers are of the form XYZ – MN – ABCD.

a. The first three digits XYZ are referred to as the <u>area number</u>. The number is associated with where you registered for your social security card. X is a number 1 – 7. Y and Z can be any number 0 – 9. Also the area code 666 has never been used. How many area code numbers are possible?

$7 \cdot 10 \cdot 10 - 1$

(7: CAN BE 1-7; 10: CAN BE 0-9; 10: CAN BE 0-9; −1: THE EXCEPTION)

699

b. The middle two digits MN are referred to as the <u>group number</u>. The group number can be any number from 01 to 99 (it cannot be 00). The last 4 digits ABCD are the serial number. The serial number can be any from 0001 to 9999 (it cannot be 0000). Additionally, the number social security number 123-45-6789 is only used in advertisements. How many social security numbers are possible?

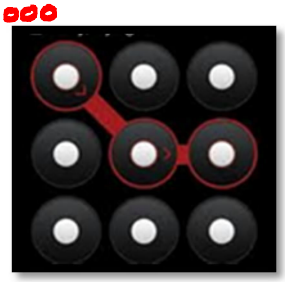
$699 \cdot 99 \cdot 9999 - 1$

(699: AREA; 99: GROUP; 9999: SERIAL)

691,940,798

PRESENT U.S. POPULATION 2015 ≈ 316,000,000

(BONUS) A person locked their android phone with a swipe code on a phone with a 3 by 3 grid block. Let's consider an example where the swipe code must start with the top left button and must pass through two more buttons. Each consecutive button is adjacent (up, down, left, right, or diagonally). How many such swipe codes are possible?



IF WE ASSUME THE FIRST BUTTON COULD BE RE-USED THEN 18 CODES

IF WE CANT RE-USE THE FIRST BUTTON THEN IT IS 15 CODES

15 OR 18 CODES